Empirical Investigation of Extreme Single-Particle Behavior of Nuclear Quadrupole Moments in Highly Collective $A \sim 150$ Superdeformed Bands

S. T. Clark,[1,*] G. Hackman,[1,†] R. V. F. Janssens,[2] R. M. Clark,[3] P. Fallon,[3] S. N. Floor,[1] G. J. Lane,[3,‡] A. O. Macchiavelli,[3] J. Norris,[1] S. J. Sanders,[1] and C. E. Svensson[3,§]

[1]*Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045*
[2]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439*
[3]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720*

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The intrinsic quadrupole moment Q_0 of superdeformed rotational bands in $A \sim 150$ nuclei depends on the associated single-particle configuration. We have derived an empirical formula based on the additivity of effective quadrupole moments of single-particle orbitals that describes existing measurements from ^{142}Sm to ^{152}Dy. To further test the formula, the predicted Q_0 moments for two superdeformed bands in ^{146}Gd of 14.05 eb were confronted with a new measurement yielding 13.9 $\pm$ 0.4 eb and 13.9 $\pm$ 0.3 eb, respectively. This excellent agreement provides empirical evidence of extreme single-particle behavior in highly deformed, collective systems.

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High-spin rotational superdeformed (SD) bands are a striking example of the interplay between macroscopic and microscopic effects in atomic nuclei [1,2]. Perturbation of single-particle levels by deformation and collective rotation provides a shell-correction energy to stabilize prolate-deformed nuclei with a major-to-minor axis ratio of 2:1 against both fission and immediate γ decay to less-deformed configurations. Nuclei in the vicinity of superdeformed "doubly magic" ^{152}Dy [1] have been studied in detail through γ-spectroscopic techniques. High-precision lifetime measurements with the Doppler-shift attenuation method (DSAM) provide information on macroscopic intrinsic quadrupole moments Q_0 with accuracies of a few percent. These Q_0 moments have been found to depend on the single-particle configuration of the SD band. In particular, measurements for ^{149,148}Gd, ^{151}Tb, and ^{151,152}Dy [3,4] have shown that bands with an identical occupation of high angular-momentum orbitals have identical Q_0 values. Furthermore, the Q_0 moment of the yrast SD band in ^{149}Gd can be obtained by deducing an effective charge quadrupole moment q_i of orbital i from differences in Q_0 between ^{152}Dy and neighboring nuclei ^{151}Tb and ^{151}Dy, then subtracting those q_i's for the holes relative to ^{152}Dy from total measured Q_0 of ^{152}Dy [3,4]. Similar additivity paradigms have been used to account for the observed dynamic moments of inertia [5] and incremental alignments [6] of these bands. The Q_0 moment measurements suggest that single-particle q_i's of orbitals near the Fermi surface are additive; that is, Q_0 can be understood as a simple algebraic sum of effective q_i moments for any given configuration.

Support for the additivity paradigm was provided by theoretical investigations of Q_0 moments near ^{152}Dy. Effective single-particle q_i moments extracted from mean-field calculations [7–9] were shown to lead to Q_0 moments that were numerically equivalent to those derived directly from detailed mean-field calculations. It was suggested in Ref. [7] that this was a consequence of "extreme shell-model" behavior in which the nucleus is described by independent, noninteracting particles in a mean field.

A lifetime measurement in ^{142}Sm [10] showed the unexpected result that the difference in Q_0 values between the excited and yrast SD bands was consistent with expectations based on q_i effective moments derived from the measurements near ^{152}Dy [3,4]. More strikingly, it was suggested that the yrast band's Q_0 moment could be understood by extracting q_i values near a ^{152}Dy core, and subtracting *ten* of these q_i values corresponding to the ten holes. A subsequent theoretical investigation with the modified harmonic oscillator approach [8] calculated the difference between Q_0 moments of ^{142}Sm and ^{152}Dy in two ways. First, the total energy of each SD band was minimized with respect to quadrupole and hexadecapole deformation, and its Q_0 moment was calculated. Second, using additivity, the Q_0 values were obtained from the q_i effective moments calculated *for the* ^{152}Dy *core*. The results of both approaches were numerically the same.

These results suggest that there is a single set of q_i effective moments that can be used to explain, or predict, the Q_0 moment of any SD band over the entire range of $A \sim 150$ nuclei, even those ten or more nucleons away from the core nucleus in which they are calculated. In other words, the intrinsic quadrupole moments of these bands may represent a better example of the applicability of extreme shell model concepts than is the case in spherical nuclei near shell closures, where additivity breaks down rapidly as one moves away from the closed shell. However, for the additivity paradigm to be considered a meaningful physical description, it must be shown that, within the uncertainties of experimental data, it can both describe *and* predict Q_0 moments. In the present work, assuming extreme shell-model additivity for effective quadrupole

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moments, we have derived an empirical formula with parameters fit to existing experimental data. The results not only qualify as a good fit to the data, but the parameters are consistent with expectations from theoretical work. The empirical formula has been more rigorously tested by confronting the predicted Q_0 moments for SD bands 1 and 2 in ^{146}Gd [11] with the results of a new high-precision lifetime measurement. The empirical prediction and the measurement agree. This shows that the additivity paradigm both describes *and predicts* Q_0 moments that are fully consistent with experiments. Hence, this work provides compelling experimental evidence that macroscopic intrinsic quadrupole moments of highly collective $A \sim 150$ SD bands are indeed described by an extreme single-particle behavior of microscopic effective quadrupole moments.

There are ~30 reported high-precision Q_0 measurements of SD bands from ^{142}Sm to ^{152}Dy, [3,4,10,12–17] corresponding to ~20 active orbitals near the $A \sim 150$ SD Fermi surface [18]. If q_i of each orbital is treated as a free parameter, there would be too many such parameters for any comparison to experimental data to be meaningful. To overcome this problem, it is proposed that the relevant orbitals can be divided into four classes. The high-j orbitals with the lowest energy due to deformation splitting constitute class A. These orbitals can be identified by their Nilsson numbers, namely the $[770]\frac{1}{2}$ neutron orbitals and $[660]\frac{1}{2}, [651]\frac{3}{2}$ proton orbitals. They are also often identified as $\nu 7_{1,2}$ and $\pi 6_{1,2,3,4}$ in the notation of Ref. [5]. Class B comprises the $N = 6$ neutron orbitals, with Nilsson numbers $[651]\frac{1}{2}$ and $[642]\frac{5}{2}$, that are responsible for the band crossings observed in, e.g., ^{146,147}Gd [19]. Class C encompasses the often-called "magic" orbitals associated with identical bands, namely $\nu[411]\frac{1}{2}$ and $\pi[301]\frac{1}{2}$ [20]. A fourth class D is considered to describe "other" orbitals, in particular proton orbitals that will be active in ^{144}Gd and ^{142}Sm. It is further proposed that all the orbitals within one class can be characterized by a single effective quadrupole moment $\overline{q}_X$. Assuming that the additivity paradigm is valid, and knowing the single-particle configurations inferred from dynamic moments of inertia, from comparison to theory, or from both, the Q_0 moment of any $A \sim 150$ SD band is given by the formula

$$Q_0[abcd] = Q_c - a \cdot \overline{q}_a - b \cdot \overline{q}_b - c \cdot \overline{q}_c - d \cdot \overline{q}_d , \tag{1}$$

where Q_c refers to the Q_0 moment of a core SD band, and $[abcd]$ are the number of holes in class A, B, C, D orbitals, respectively, relative to that core. Q_c and each $\overline{q}_{a,b,c,d}$ are independent free parameters to be determined from experimental data. As is traditional, ^{152}Dy has been selected as the core.

The parameters of Eq. (1) were fit to 23 bands that have been measured with high precision (Table I). For each band the $[abcd]$ numbers were derived from the configurations discussed in the reference given in Table I. For ^{144}Gd, the Q_0 values both above and below the backbend were included as separate data points [13]. In other publications it was noted that even when analyses used different parameters for stopping powers and side feeding, the Q_0 values deduced from separate experiments were in excellent agreement [4,16]. Systematic errors are neglected as they would introduce an overall scaling factor to the Q_c and $\overline{q}_{a,b,c,d}$ parameters, which would compromise neither the validity nor the predictive power of Eq. (1).

TABLE I. Q_0 moment measurements and Eq. (1) expectations for $A \sim 150$ SD bands. Configurations are represented as number of class A, B, C, and D holes relative to ^{152}Dy.

Band		$[abcd]$ Configuration	Ref.	Q_0 (eb) Measured	Q_0 (eb) Eq. (1)
^{142}Sm	a	[5401]	[10]	11.7 ± 0.1	11.69
	b	[3340]	[10]	13.2 ± 0.8	13.49
^{143}Eu	1	[5400]	[12]	13.0 ± 1.5	11.89
^{144}Gd	1	[4400]	[13]	$13.7^{+1.1}_{-0.9}$	12.81
	1a	[640-2]	[13]	$11.6^{+1.4}_{-1.0}$	11.37
^{148}Gd	1	[3100]	[3]	14.6 ± 0.2	14.69
	2	[3100]	[3]	14.8 ± 0.3	14.69
	5	[0040]	[3]	17.8 ± 1.3	17.21
^{149}Gd	a	[3000]	[3]	15.0 ± 0.2	15.01
	b	[2100]	[3]	15.6 ± 0.3	15.61
	c	[2010]	[3]	15.2 ± 0.5	15.79
	d	[0030]	[3]	17.5 ± 0.6	17.35
^{149}Tb	1	[2100]	[15]	15.3 ± 0.2	15.61
	2	[2100]	[15]	$15.8^{+0.4}_{-0.3}$	15.61
	3	[1110]	[15]	$16.4^{+0.3}_{-0.4}$	16.39
	4	[2100]	[15]	$16.0^{+0.6}_{-0.5}$	15.61
^{150}Gd	1	[2000]	[17]	$17.0^{+0.5}_{-0.4}$	15.93
	2	[1010]	[17]	$17.4^{+0.5}_{-0.4}$	16.71
	5	[0020]	[17]	16.8 ± 1.2	17.49
^{151}Tb	1	[1000]	[4]	$16.8^{+0.7}_{-0.6}$	16.85
^{151}Dy	1	[1000]	[4]	$16.9^{+0.2}_{-0.3}$	16.85
	4	[0010]	[4]	$17.5^{+1.1}_{-0.7}$	17.63
^{152}Dy	1	[0000]	[3]	17.5 ± 0.2	17.77
^{146}Gd	1	[3300]	This work	13.9 ± 0.4	14.05
	2	[3300]	This work	13.9 ± 0.3	14.05

Some data had to be excluded from the fit for the following reasons. Band 6 of ^{150}Gd was interpreted as a collective excitation of the SD shape [17] and, therefore, should not be described by Eq. (1). Configuration assignments for bands 2 and 3 of ^{151}Dy [4] and 3 and 4 of ^{150}Gd [17] remain a matter of discussion. The ^{145}Gd configurations were suggested following a lifetime measurement [14]; these data were excluded to avoid circular reasoning. Recent ^{151}Tb measurements used a different model for the decay-out probabilities [16] whose effects were not considered in as much detail as stopping power and side feeding. (The result for band 1 agrees within errors with the prior measurement [4].)

A least-squares minimization to the data gives a goodness-of-fit value $\chi^2_{\min}/n = 0.90$ for the parameters

TABLE II. Best-fit values for quadrupole moment parameters, compared to reported values from Hartree-Fock ($q_{\rm eff}^{\rm SkP}$, Ref. [7]) and modified harmonic oscillator (MHO) ($q_{\rm eff}^{\rm mic}$, Ref. [8]) calculations, all values in eb. No values were reported for class D orbitals in Ref. [7].

Parameter	Fit	HF	MHO
Q_c	17.77 ± 0.23	18.49	18.10
$\overline{q}_a$	0.92 ± 0.11	0.75	0.85
$\overline{q}_b$	0.32 ± 0.12	0.33	0.32
$\overline{q}_c$	0.14 ± 0.13	0.00	−0.24
$\overline{q}_d$	0.20 ± 0.31		0.81

listed in Table II. Uncertainties have been obtained by inspecting the extents of the volume enclosed by $\chi^2_{\rm min} + 1$ in the five-dimensional parameter space. For comparison, theoretical values extracted from Refs. [7,8] are also tabulated. The theoretical values for $\overline{q}_a$ and $\overline{q}_b$ compare quite well. The class C orbitals have $\overline{q}_c \sim 0$, which is interesting in light of the fact that they are also expected to make no contribution to the moment of inertia of an SD band [20]. Since $\overline{q}_d$ averages over a number of possibly quite different orbitals, no conclusions on class D can be drawn; the only calculated values available are for the $[541]\frac{1}{2}$ and $[523]\frac{5}{2}$ orbitals [8].

Figure 1 shows the correlation between Eq. (1) and measured Q_0 moments. Clearly the measured data are well described by Eq. (1), as one should expect for $\chi^2_{\rm min}/n \sim 1$. However, a more rigorous test is to ascertain its predictive power for an unmeasured or poorly measured SD band in this mass region. The first two SD bands of ^{146}Gd [11] have been selected for a DSAM lifetime measurement, since they share a configuration in terms of $[abcd]$ parameters [3300] that has not been measured with high accuracy [19]. Equation (1) predicts a Q_0 moment of 14.05 eb for both bands.

The experiment was performed at the 88-inch cyclotron at Lawrence Berkeley National Laboratory. A 155 MeV ^{29}Si beam was directed upon a 0.96 mg/cm^2 isotopically enriched ^{122}Sn target evaporated on a 15 mg/cm^2 Au substrate with a 70 μg/cm^2 Al buffer layer between them to prevent alloying. SD states in ^{146}Gd were populated via the $5n$ fusion-evaporation channel at an estimated mean recoil velocity as a fraction of the speed of light of $\beta_0 = v_0/c = 2.04\%$. Gamma rays were detected with Gammasphere [21], comprising 100 Compton-suppressed high-purity Ge (HPGe) detectors fitted with 1.0 mm thick Pb absorbers. The in-beam event trigger required the presence of five or more Compton-suppressed HPGe detectors in prompt coincidence. A total of 1.5×10^9 events were written to tape. Spectra were generated by double gating on triple-coincidence data or triple gating on quadruple-coincidence data. Figure 2 illustrates the quality of the background-subtracted coincidence data.

The fractional Doppler shifts of each of the lines in the SD bands are plotted in Fig. 3. These have been compared to values calculated over a grid of Q_0 and side-feeding quadrupole moments $Q_{\rm sf}$ with a cascade of five side-feeding transitions, using the FITFTAU code [22] and the SRIM2000 stopping powers [23]. Again, the χ^2 surface was inspected to determine the best-fit values and their uncertainties.

The results for bands 1 and 2 are $Q_0 = 13.9 \pm 0.4$ eb and 13.9 ± 0.3 eb with $\chi^2/n = 0.9$ and $\chi^2/n = 1.4$, respectively. These measured Q_0 values are in excellent agreement with the predicted value of 14.05 eb. Thus, Eq. (1) has predictive power. Therefore, within the limits of the available experimental data, this work demonstrates *empirically* that the quadrupole-moment additivity paradigm is valid, and hence that the macroscopic intrinsic

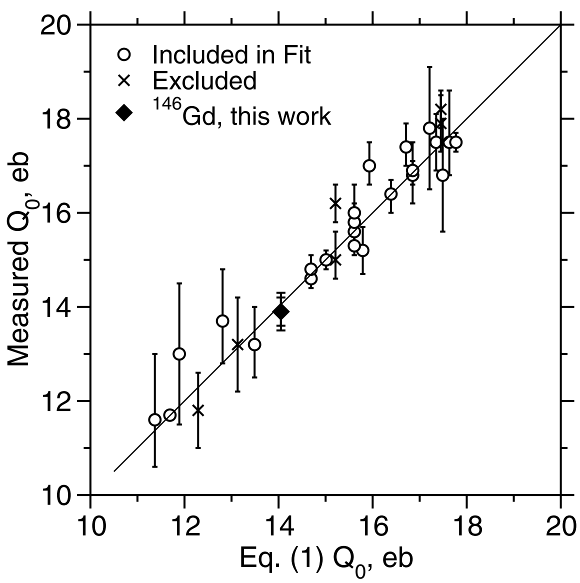

FIG. 1. Q_0 moments, measured vs Eq. (1), for $A \sim 150$ SD bands. The diagonal line represents where data points would lie if measurement and Eq. (1) were equal.

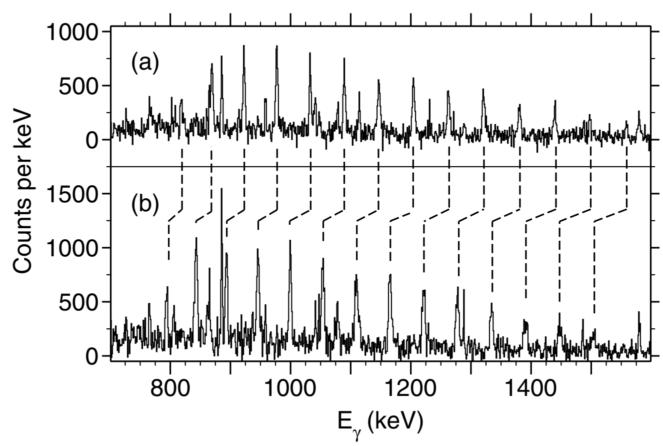

FIG. 2. Partial spectra of γ rays double gated by nonidentical pairs of transitions in ^{146}Gd SD band 2 observed in (a) forward detectors at an average angle of $\overline{\theta} = 41°$; and, (b) backward detectors, $\overline{\theta} = 142°$. Dashed lines connect Doppler-shifted γ-ray lines from the same SD transition.

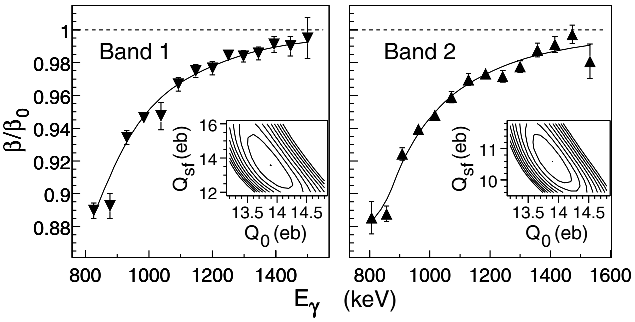

FIG. 3. Fractional Doppler shifts of ^{146}Gd SD band transitions. Curves indicate the best fit to the data. Insets show absolute χ^2 surfaces as functions of Q_0 and Q_{sf}. Dots in the centers indicate χ^2 minima; surrounding contours are spaced in intervals of one unit.

charge quadrupole moments of collective superdeformed rotational bands from ^{142}Sm to ^{152}Dy exhibit extreme microscopic single-particle behavior.

This conclusion raises some interesting issues. Reference [2] proposes that there should be no physics distinction between the SD bands along a chain from $A \sim 130$ to $A \sim 150$. A recent systematic study of Q_0 moments in the $A \sim 130$ mass region [24] suggests that it may be possible to derive a formula similar to Eq. (1) valid within that region. It should also be noted that Eq. (1) is not sensitive to the choice of core; the analysis has been repeated with ^{142}Sm as the core, and the best-fit values of $\overline{q}_{a,b,c,d}$ are the same. One may then conjecture that there may be a unique set of effective quadrupole moments q_i that can describe or predict Q_0 moments of ~150 SD bands spanning a ~20-unit range of masses. This could be demonstrated by deriving an expression such as Eq. (1) for the entire $A \sim 130-150$ region, with a small number of extra $\overline{q}_X$ parameters accounting for the broader range of active orbitals. It will be necessary, then, for both experiment and theory to turn their attention to high-precision measurements and detailed calculations near ^{142}Sm to bridge the gap between $A \sim 130$ and $A \sim 150$. In addition, within the $A \sim 150$ region, Eq. (1) with the parameters of Table II can in fact be used to guide configuration assignments to new SD bands. Finally, the additivity paradigm should be tested in the SD bands of $A \sim 36$ nuclei where there are fewer active orbitals and full shell-model calculations are tractable [25], although there are technical obstacles to be overcome to obtain the needed accuracy in Q_0 measurements.

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*Present address: Physics Department, Boston University, Boston, MA 02215.
†Present address: TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada.
‡Present address: Department of Nuclear Physics, Australia National University, Canberra, ACT 0200, Australia.
§Present address: Department of Physics, University of Guelph, Guelph, ON N1G 2W1, Canada.

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